

Inverse

Exercise 2.14

a. $A = \begin{pmatrix} -1 & -1 & -3 \\ 1 & 2 & 2 \\ 0 & 5 & -4 \end{pmatrix}$

b. $B = \begin{pmatrix} 1 & 3 & 3 \\ -1 & -4 & -4 \\ -1 & 3 & 2 \end{pmatrix}$

c. $C = \begin{pmatrix} -1 & -3 & -4 \\ 1 & 4 & -3 \\ 1 & 4 & -4 \end{pmatrix}$

d. $D = \begin{pmatrix} 0 & 1 & -4 \\ -1 & -1 & 1 \\ -2 & -1 & -1 \end{pmatrix}$

e. $E = \begin{pmatrix} 1 & 1 & 1 \\ -3 & -4 & -1 \\ 2 & 4 & -3 \end{pmatrix}$

f. $F = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ 1 & 0 & 1 \end{pmatrix}$

Solution Exercise 2.14

a.

$$A = \begin{pmatrix} -1 & -1 & -3 \\ 1 & 2 & 2 \\ 0 & 5 & -4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} -1 & -1 & -3 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-1 R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-1 R_1 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\frac{1}{5} R_3 \leftrightarrow R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 1 & -1 & 1 & 1 & 0 \end{array} \right) \xrightarrow{-1 R_2 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & -\frac{1}{5} & 1 & 1 & -\frac{1}{5} \end{array} \right) \xrightarrow{-1 R_2 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{3\frac{4}{5}}{5} & -1 & 0 & -\frac{1}{5} \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & -\frac{1}{5} & 1 & 1 & -\frac{1}{5} \end{array} \right) \xrightarrow{-5 R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{3\frac{4}{5}}{5} & -1 & 0 & -\frac{1}{5} \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right) \xrightarrow{-3\frac{4}{5} R_3 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 18 & 19 & -4 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right) \xrightarrow{\frac{4}{5} R_3 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 18 & 19 & -4 \\ 0 & 1 & 0 & -4 & -4 & 1 \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right)$$

$$\text{inv } A = \begin{pmatrix} 18 & 19 & -4 \\ -4 & -4 & 1 \\ -5 & -5 & 1 \end{pmatrix}$$

b.

$$B = \begin{pmatrix} 1 & 3 & 3 \\ -1 & -4 & -4 \\ -1 & 3 & 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ -1 & -4 & -4 & 0 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \stackrel{1R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ -1 & -4 & -4 & 0 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \stackrel{1R_1 \rightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \stackrel{1R_1 \rightarrow R_3}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 6 & 5 & 1 & 0 & 1 \end{array} \right) \stackrel{\frac{1}{6}R_3 \leftrightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & -1 & -1 & 1 & 1 & 0 \end{array} \right) \stackrel{1R_2 \rightarrow R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & -\frac{1}{6} & 1\frac{1}{6} & 1 & \frac{1}{6} \end{array} \right) \stackrel{-3R_2 \rightarrow R_1}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & -\frac{1}{6} & 1\frac{1}{6} & 1 & \frac{1}{6} \end{array} \right) \stackrel{-6R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right) \stackrel{-\frac{1}{2}R_3 \rightarrow R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 3 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right) \stackrel{-\frac{5}{6}R_3 \rightarrow R_2}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 3 & 0 \\ 0 & 1 & 0 & 6 & 5 & 1 \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right)$$

$$\text{inv } B = \begin{pmatrix} 4 & 3 & 0 \\ 6 & 5 & 1 \\ -7 & -6 & -1 \end{pmatrix}$$

c.

$$C = \begin{pmatrix} -1 & -3 & -4 \\ 1 & 4 & -3 \\ 1 & 4 & -4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} -1 & -3 & -4 & 1 & 0 & 0 \\ 1 & 4 & -3 & 0 & 1 & 0 \\ 1 & 4 & -4 & 0 & 0 & 1 \end{array} \right) \stackrel{-1R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 1 & 4 & -3 & 0 & 1 & 0 \\ 1 & 4 & -4 & 0 & 0 & 1 \end{array} \right) \stackrel{-1R_1 \rightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 1 & 4 & -4 & 0 & 0 & 1 \end{array} \right) \stackrel{-1R_1 \rightarrow R_3}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 1 & -8 & 1 & 0 & 1 \end{array} \right) \stackrel{1R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 1 & -8 & 1 & 0 & 1 \end{array} \right) \stackrel{-1R_2 \rightarrow R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array} \right) \stackrel{-3R_2 \rightarrow R_1}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 25 & -4 & -3 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array} \right) \stackrel{-1R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 25 & -4 & -3 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right) \stackrel{-25R_3 \rightarrow R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -28 & 25 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right) \stackrel{7R_3 \rightarrow R_2}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -28 & 25 \\ 0 & 1 & 0 & 1 & 8 & -7 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right)$$

$$\text{inv } C = \begin{pmatrix} -4 & -28 & 25 \\ 1 & 8 & -7 \\ 0 & 1 & -1 \end{pmatrix}$$

d.

$$D = \begin{pmatrix} 0 & 1 & -4 \\ -1 & -1 & 1 \\ -2 & -1 & -1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 0 & 1 & -4 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 1 & 0 \\ -2 & -1 & -1 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-\frac{1}{2}R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ -1 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 1 & 0 & 0 \end{array} \right) \xrightarrow[\sim]{1R_1 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \end{array} \right) \xrightarrow[\sim]{1R_3 \leftrightarrow R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 1 & -\frac{1}{2} \end{array} \right) \xrightarrow[\sim]{\frac{1}{2}R_2 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 1 & -\frac{1}{2} \end{array} \right) \xrightarrow[\sim]{-\frac{1}{2}R_2 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 2\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 1 & -\frac{1}{2} \end{array} \right) \xrightarrow[\sim]{-2R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array} \right) \xrightarrow[\sim]{-2\frac{1}{2}R_3 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 5 & -3 \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array} \right) \xrightarrow[\sim]{4R_3 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 5 & -3 \\ 0 & 1 & 0 & -3 & -8 & 4 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array} \right)$$

$$\text{inv } D = \begin{pmatrix} 2 & 5 & -3 \\ -3 & -8 & 4 \\ -1 & -2 & 1 \end{pmatrix}$$

e.

$$E = \begin{pmatrix} 1 & 1 & 1 \\ -3 & -4 & -1 \\ 2 & 4 & -3 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ -3 & -4 & -1 & 0 & 1 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-\frac{1}{3}R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-1R_1 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-2R_1 \rightarrow R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 \\ 0 & 1\frac{1}{3} & -3\frac{2}{3} & 0 & \frac{2}{3} & 1 \end{array} \right) \xrightarrow[\sim]{\frac{3}{4}R_3 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 \end{array} \right) \xrightarrow[\sim]{\frac{1}{3}R_2 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & -\frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{4} \end{array} \right) \xrightarrow[\sim]{-1\frac{1}{3}R_2 \rightarrow R_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & -1 & -1 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & -\frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{4} \end{array} \right) \xrightarrow[\sim]{-4R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & -1 & -1 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right) \xrightarrow[\sim]{-4R_3 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & 7 & 3 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right) \xrightarrow[\sim]{2\frac{3}{4}R_3 \rightarrow R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & 7 & 3 \\ 0 & 1 & 0 & -11 & -5 & -2 \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right)$$

$$\text{inv } E = \begin{pmatrix} 16 & 7 & 3 \\ -11 & -5 & -2 \\ -4 & -2 & -1 \end{pmatrix}$$

f.

$$F = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 3 & -5 & -5 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \stackrel{\frac{1}{3}R_2 \leftrightarrow R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 1 & -2 & -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \stackrel{-1 R_1 \rightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \stackrel{-1 R_1 \rightarrow R_3}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \\ 0 & 1\frac{2}{3} & 2\frac{2}{3} & 0 & -\frac{1}{3} & 1 \end{array} \right) \stackrel{\frac{5}{3}R_3 \leftrightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \end{array} \right) \stackrel{\frac{1}{3}R_2 \rightarrow R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & \frac{1}{5} & 1 & -\frac{2}{5} & \frac{1}{5} \end{array} \right) \stackrel{1\frac{2}{3}R_2 \rightarrow R_1}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & \frac{1}{5} & 1 & -\frac{2}{5} & \frac{1}{5} \end{array} \right) \stackrel{5R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \stackrel{-1 R_3 \rightarrow R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 2 & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \stackrel{-1\frac{3}{5}R_3 \rightarrow R_2}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 2 & 0 \\ 0 & 1 & 0 & -8 & 3 & -1 \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right)$$

$$\text{inv } F = \begin{pmatrix} -5 & 2 & 0 \\ -8 & 3 & -1 \\ 5 & -2 & 1 \end{pmatrix}$$